Minton ,
choreographies.
check.

Course Overview

1. Lagrangian systems. Natural Mechanical systems. Symmetry vs topological constraints. Works of: Gordon, Poincaré, me. Braids. Planarity. Strong vs weak forces.

2. Shape space, shape sphere, reduction. The Eight

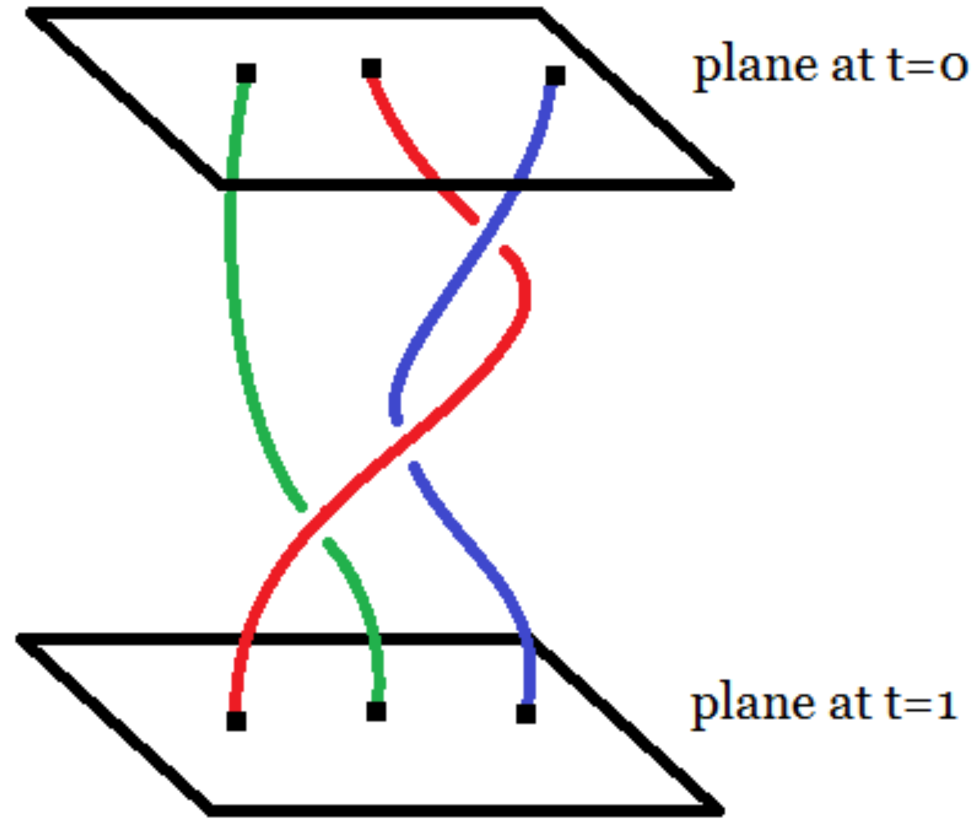
3. Functional analysis for the direct method. Two point boundary value problem. The eight again. time permitting: Marchall.

4. Infinitely many syzygies and coplanarities. Riem. geom methods.

5. Open ending. Open problems.

¿ Why (in 2022) study the N-body problem ?

3. Is every braid on N strands realized by some sol'n?



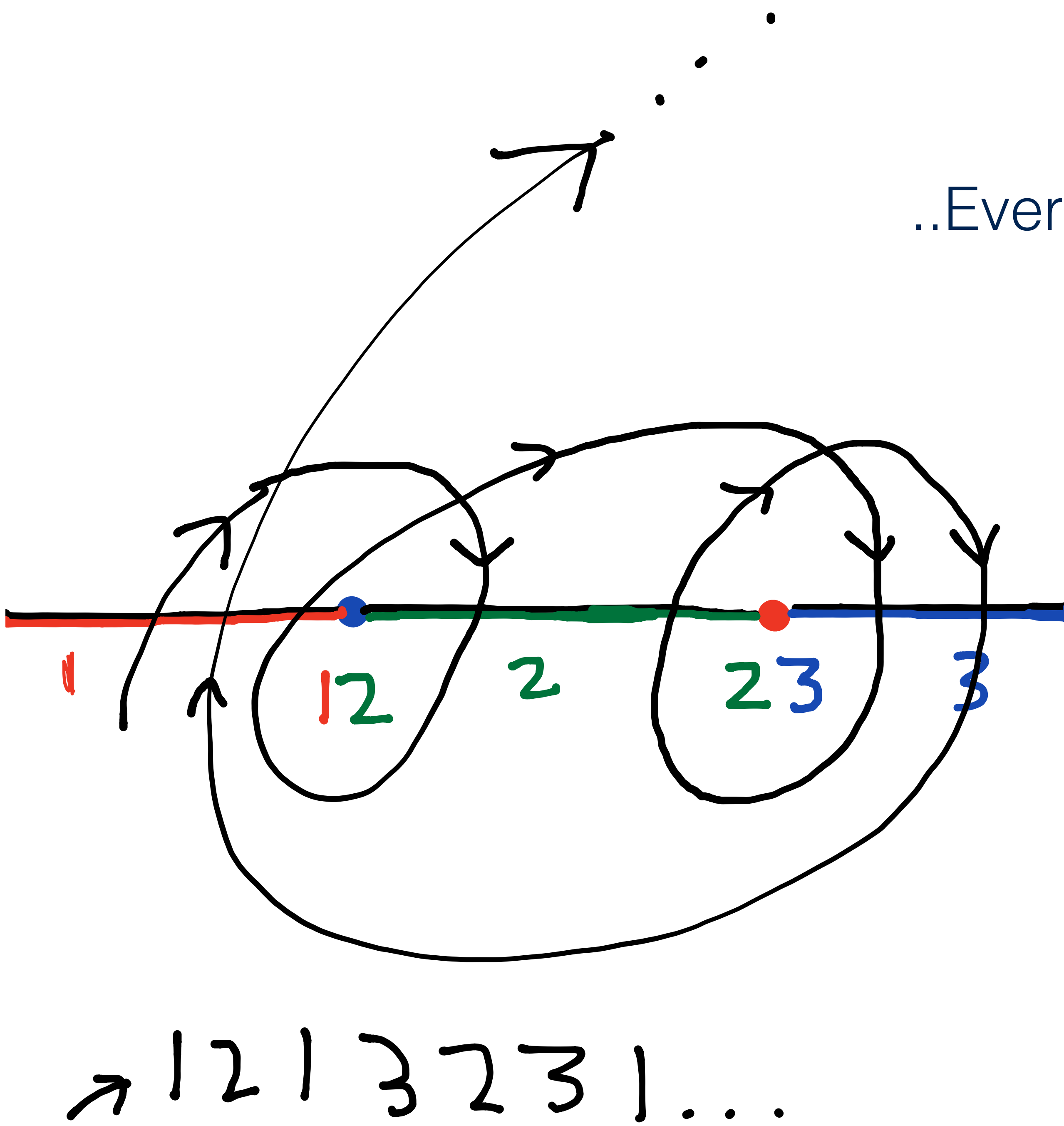
A solution for which

$$q_a(t+T) = q_{\sigma(a)}(t) : \sigma : \{1, \ldots, N\} \to \{1, \ldots, N\}$$

represents a *braid:* **an element in the fund. group of**

$$((\mathbb{R})^2)^N \setminus \text{ collisions})/S_N$$

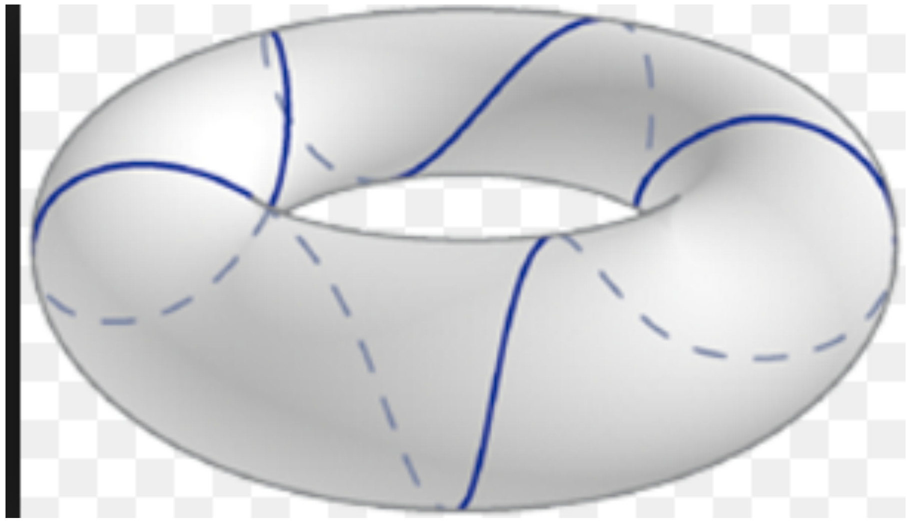
the config. space of N identical distinct points in the plane.



..Every free homotopy class…

Figure 8: 123123

Thm.
In a *compact* Riemannian geometry
every free homotopy class of loops
is realized by a periodic *geodesic*.



Pf. Direct method of the calculus of var'ns. Minimize length of loops over all loops which represent the given class

3-body. A conjugacy class in the pure braid group on 3 strands
=
a free homotopy class of loops in the ***collision-free*** planar 3-body config . space

SUR LES SOLUTIONS PÉRIODIQUES ET LE PRINCIPE DE MOINDRE ACTION

Comptes rendus de l'Académie des Sciences, t. 123, p. 915-918 (30 novembre 1896).

La théorie des solutions périodiques peut, dans certains cas, se rattacher au principe de moindre action.

Supposons trois corps se mouvant dans un plan et s'attirant en raison inverse du cube des distances ou d'une puissance plus élevée de ces distances; j'appelle a, b, c ces trois corps.

The N-body problem, the braid group, and action-minimizing periodic solutions.

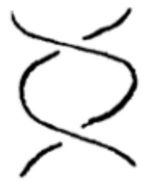
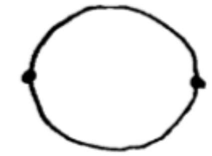
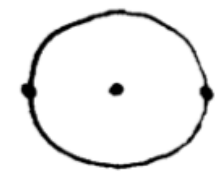
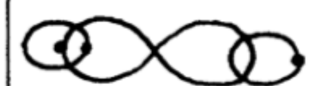
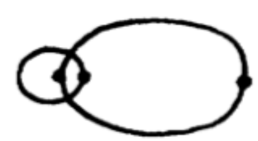
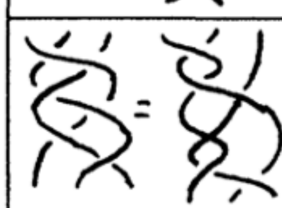
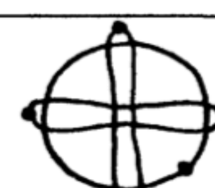
PHYSICAL REVIEW
LETTERS

OLUME 70 14 JUNE 1993

Braids in Classical Dynamics

Cristopher Moore
Santa Fe Institute, Santa Fe, New Mexico 87501
(Received 19 October 1992)

Point masses moving in 2+1 dimensions draw out braids in space-time. If they move under
influence of some pairwise potential, what braid types are possible? By starting with fictional path
the desired topology and "relaxing" them by minimizing the action, we explore the braid types of po
tials of the form $V \propto r^{\alpha}$ from $\alpha \leq -2$, where all braid types occur, to $\alpha = 2$, where the system is int
able. We also discuss issues of symmetry and stability. We propose this kind of topological classifica
as a tool for extending the "symbolic dynamics" approach to many-body dynamics.

braid	b_i	orbit	existence
	b_1^2		exists for all α
	—		exists for all α
	$b_1^2 b_2^2$		$\alpha < -1.1 \pm 0.05$
	$b_1^2 b_2^{-2}$		$\alpha < -1.4 \pm 0.05$
	$(b_1 b_2)^3$		exists for all α
	$(b_1 b_2^{-1})^3$		$\alpha < 2$
	$(b_1^2 b_2)^2$		$\alpha < -1.0 \pm 0.05$
	$(b_1^2 b_2^{-1})^2$		$\alpha < -1.7 \pm 0.05$
	$b_1 b_2 b_1^{-1} b_2 b_1 b_2^{-1}$ $= b_1^2 b_2 b_1^{-2} b_2$		at least $\alpha \leq 2$

FINI