A correction/clarification

someone asked

`Is the shape sphere and shape space covered in the references''

Yes. sec 0.4, p 46- 50

Recall:

Course Overview

1. Lagrangian systems. Natural Mechanical systems. Symmetry vs topological constraints. Works of: Gordon, Poincaré, me. Braids. Planarity. Strong vs weak forces.

2. Shape space, shape sphere, reduction. The Eight

3. Functional analysis for the direct method.Two point boundary value problem. The eight again. time permitting: Marchall.

4. Infinitely many syzygies and coplanarities. Riem. geom methods.

5. Open ending. Open problems.

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$$\ddot{q} = \nabla U(q) \tag{1}$$

Given: $q_0, q_1 \in \mathbb{E}$ and a time $T > 0$.

Find a solution $q(t)$ satisfying $q(0) = q_0, q(T) = q_1$

Solution: Direct method of the calculus of variations

$$\mathrm{L}\ (\mathrm{q},\ \mathrm{v}) = \mathrm{K}(\mathrm{v}) + \mathrm{U}(\mathrm{q})$$

$$A(q(\cdot)) = \int_0^T L(q(t), \dot{q}(t))dt$$

Problem Minimize the action $A(q(\cdot))$ over all paths $q : [0, T] \to \mathbb{E}$ such that $q(0) = q_0$ and $q_1 = q(T)$.

If the minimum ***exists*** and is sufficiently smooth, it solves our two-point boundary value problem

To get existence:

$$\Omega = \text{space of absolutely continuous paths } q : [0, T] \to \mathbb{E}$$

$$\text{having } q(0) = q_0, q(T) = q_1 \text{ and } A(q) < \infty$$

and set:

$$a = \inf_{c \in \Omega} A(c).$$

so that there exists :

$$c_n \in \Omega \text{ such that } \lim_{n \to \infty} A(c_n) = a.$$

We construct the minimizer c_* as a limit of a subsequence of the c_n

STEPS.
Show:

STEP 1. A subsequence of the c_n converges uniformly to a continuous c_*.

STEP 2. $c_* \in \Omega$.

STEP 3. $A(c_*) = a$.

STEP 4. A is differentiable.

$dA(c_*)(h) = 0$ for all variations h tangent to Ω.

STEP 5. This minimizer c_* satisfies Newton's equations.

Overview of how the steps are done

STEP 1. Arzela-Ascoli

STEP 2. Banach-Alaoglu. Notion of weak compactness.
Work in Sobolev space H^1, a Hilbert space closely related to Ω.

STEP 3. Ideas from steps 1 and 2.

STEP 4. first quarter calculus, really

STEP 5. Fundamental theorem of the calculus of variations.

DETAILS:

on the Board!