MAKE-UP PROBLEM FOR HW 5

A. Redo exercise 4, but with $\mathbb{Z}_2$ replaced by $\mathbb{Z}_k$, $k > 2$ [1] Here $\mathbb{Z}_k$ is the cyclic group of integers mod k. Thus, your vertex set will be $V = (\mathbb{Z}_k)^n$. Your edge set is defined by declaring vertices to be adjacent if and only if exactly one of their n coordinates are different. [2]

Show that" translations" are automorphisms of this graph. Thus, for $v \in V$ show that the "translation by v map": $T_v : V \to V$ which is defined by $T_v(s) = s + v$ is a graph automorphism [3]

B. [EXTRA PRACTICE PROBLEM] Let S be a set of n elements. Fix an integer $k < n$. Form a simple graph Γ from S whose vertex set $V = V(\Gamma)$ is the set of all k element subsets of S. Declare that two vertices are adjacent if and only if the corresponding subsets of S are disjoint. Let $G = S_n$ be the group of all permutations of S [4] i) Describe how a $\sigma \in G$ acts on V. ii) Prove that this action of σ is a graph automorphism.

---

[1] In bio-informatics $k = 4$ corresponding to the 4 letters, A, C, G, T of the genetic code.

[2] In symbols, $v, w \in V$ are adjacent if there is an index i_0 in the index set $\{1, 2, \ldots, n\}$ such that $v_{i_0} \neq w_{i_0}$ while for all $j \neq i_0$ $v_j = w_j$.

[3] s and v each have n components, $s_i, v_i \in \mathbb{Z}_k, i = 1, \ldots, n$. Their sum $s + v \in V$ is the vector whose ith component is $s_i + v_i$, the addition being mod k.

[4] a permutation of a finite set is just an invertible map from the set onto itself.