Dynamical Systems circle maps HW

Connor Tiedeman

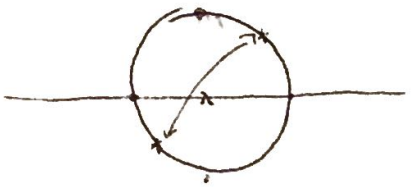
1. Show $\rho: \mathrm{Homeo}_+(S^1) \longrightarrow S^1$ is not a group homeo.

$PSL_2(\mathbb{R})$ acts on $\mathbb{R} \cup \{\infty\} = \mathbb{RP}^1 = S^1$ by $\begin{pmatrix} a & b \\ c & d \end{pmatrix} x = \frac{ax+b}{cx+d}$

$f: x \mapsto x+1$ on $\mathbb{R} \cup \{\infty\}$ gives a circle homeo with 1 fixed pt. (∞)

So has rotation # $\underline{0}$.

$g: x \mapsto -\frac{1}{x}$ gives a circle homeo with every pt. of period 2

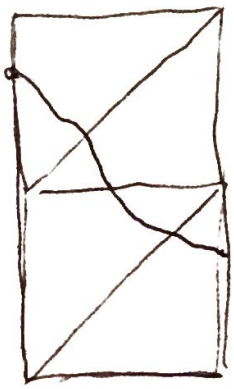


So has rotation # $\underline{\frac{1}{2}}$.

however, $f \circ g: x \longmapsto -\frac{1}{1+x}$ has every pt. of period 3

so has rotation # $\frac{1}{3} \neq \frac{1}{2}$ to rotate.

2. True, in fact each orientation reversing homeo has exactly 2 fixed pts. This is because its lift has negative slope:



3. Is also true by same picture. (the lift must be decreasing).