Syllabus, for Dynamical Systems. Math 235, Winter 04.

The class is in lecture format with HW. Students will present some solutions on the board.

The texts used are GH= Guckenheimer and Holmes: Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields. RM= Ratiu and Marsden: Manifolds, Tensor Analysis, and Applications. KH= Katok and Hassenblat: Introduction to the Modern Theory of Dynamical Systems. A1 = Arnol'd: Geometrical Methods in the theory of Ordinary Differential Equations. A2 = Arnol'd: Mechanics. Ruelle. Chenciner: in Encylopaedia Universalis, Systemes Dynamiques Differentiables.

with GH and KH being the main sources.

Lecture 1. Reading: RM p. 196. KH. Section 0.

In-class diagnostic quiz for evaluating level of students. Mostly basic ODE.

Overview of Dynamical Systems. ODEs and vector fields on manifolds. Flows. The straightening lemma: any two nonzero vector fields on spaces of the same dimension are locally diffeomorphic. Complete vs. incomplete flows.

break.

Maps.

Poincare recurrence lemma.

Generalities: on the main branches of dynamical systems. Ergodic theory (impose a measure: Birkhoff; Boltzman..). Topological Dynamics (impose topology on space; Poincare). Smooth Dynamics (impose a smooth structure...). Hamiltonian dynamics (Poincare; here: facutly at UCSC: Ginburg, Lewis, Montgomery). Hyperbolic theory (Anosov, Smale...). Complex dynamics (Mandelbrot; Milnor; McMullen). Actions of (large) infinite discrete groups. Normal forms.

WED. Jan 7. NO LECTURE

2. Reading: GH 1.1-1.3. and sec. 1.2. RM. KH sec. 1.2 and also p. 10-11, under 'Hartman-Grobman' in index.

Straightening lemma. Local invariants of v-fields: fixed points. Linearization at a fixed point.

Linearization for maps vs flows.

Theorems: Hyperbolicity. Hartman-Grobman theorem (statement).

BACKGROUND. Solving linear homogeneous systems.

3. Reading: GH: 1.5. Katok sec 0.3; Jan 12 (Mon). Periodic orbits. Poincare section again. Limit cycles.

Application: Van der Pol.

4. Poincare-Bendixson theorem.

5. Poincare-Hopf theorem. (Euler characteristic as a sum of indices.)

6. Gradient flows. Basics of Morse theory.

6. Averaging.

7.

Lecture 5. Doubling maps. First eg of chaotic dynamics.

Lecture 5. Eg. Twist maps. Typical behaviour. Pictures from Meese' review.

Topology: FINDING zeros of a v-fld: Poincare-Hopf theorem on Euler characteristic. FINDING fixed points of a map. Lefshetz fixed pt theorem. ... state Arnol'd conjec.

Lect 4. Reading: GH: 1.4. Katok, p. 12, sec. 6.2. The Stable and unstable manifolds of a fixed pt. 'Hadamard-Perron theorem'. Sketch Pf.

Lect. 5. READING: Katok, p. 82, and look up 'horseshoe' in the index.

Finding horseshoes when the stable and unstable mfds intersect.

Lect 6. GH: 2.2. Guckenheimer and Holmes. Some examples. 1 d. of freedom Hamiltonian systems. The pendulum. The forced damped pendulum: $1\frac{1}{2}$ degrees of freedom.

Lect 7. Returning to more examples, more behavior. 1. Doubling map.

TOPICS to be hopefully covered. In reality, there are too many here.

CONCEPTS. Fixed points. Linearization. Invariant manifolds. Suspension. Poincare section. ω and α limits. Recurrence. Conjugacy. Structural stability. Homoclinic and Heteroclinic intersections and tangles. Symbolic dynamics. Hyperbolicity. Axiom A systems. Bifurcation theory.

THEOREMS. Normal form theorems: Hartman's, Sternberg's, Poincare-Dulac. Invariant manifold theorems: Stable and Unstable and Center manifold theorems. Counting invariant manifold theorems: Lefshetz fixed point theorem. Ergodic theorem. Circle diffeo theorems: Denjoy. Structural stability theorem: Anosov.

EXAMPLES. linear systems. Circle diffeos. Linear flows on tori. The cat (or Anosov) map of T^2. Doubling maps. Contraction mappings. Gradient flows. Isometries. Projective transformations. Newton's equations. 2 and N-body equations. Geodesic flow on a negatively curved surface. On a negatively curved manifold. Horocycle flow. Smale Horseshoe.

Lectures

(*) Depending on preparation of students, some lectures may have to be devoted to differentiable manifolds, analysis thereon, or ODEs.