Syllabus, Dynamical Systems, Math 235. Fall, 2019.

Prof. Richard Montgomery; rmont@ucsc.edu

class : Tu, Th 9:50 to 11:25.

office hours: Tu: 2-4, Th : 5:30 - 6:30

web site: http://people.ucsc.edu/ rmont/classes/DynSys/2019/

Primary Texts. (a) Clarke Robinson, Dynamical Systems.

(b) M Brin and G Stuck, Introduction to Dynamical Systems.

(c) Guckenheimer & Holmes, Nonlinear Oscillations, Dynamical Systems ...

Other highly recommended texts: Hirsch and Smale and Devaney, (1974), Differential Equations, Dynamical Systems, and Linear Algebra, (Academic Press, New York)

Dynamical Systems II, in Encyc of Mathematical Sciences vol 2, ed. Sinai, Springer.

Arnold, V. I. (1978). Mathematical Methods of Classical Mechanics. New York, Springer.

Arnol'd , Avez, 'Ergodic Theory'

Halmos, 'Ergodic Theory'.

Moser: Stable and Random Motion.

Discrete Dynamical Systems, lectures in Tsinghua, 2017, A Chenciner,

url: https://perso.imcce.fr/alain-chenciner/DDS.pdf

Good as an encyclopaedia: Hasselblatt, B. and A. Katok (2003). A First Course in Dynamics: with a Panorama of Recent Developments. Cambridge, Cambridge Univ. Press.

Evaluations and grades :

50 percent HW, 40 percent in-class presentation, 10 percent class participation.

HW: You are encouraged to work together on homework assignments, however, the final write-up of each problem must be your own.

In-Class Presentations: Each student will give at least one presentation in class. I will post a list of of possible papers and topics for presentations. You can also take topics from any of the above sources or choose your own topic. Get my okay ahead of time.

Bare Bones Calendar.

Sept 26. 1st class. Nov 28. Thanksgiving Holiday. Dec 5. Last class.

Topics covered. *This list is ambitious. As you go further down the list, topics becomes less likely to be covered.* Basic Examples and Definitions. ODEs: Basic theory: "Well-posedness". Straightening lemma. N-body problem. "Oldest problem in dynamical systems". Systems arising from mechanics. From geometry. Fixed points. Periodic orbits. Linear theory. Linearization. Flows vs. Maps. Poincare Return Map. Suspension. Conjugation and semi-conjugation. Normal forms. Circle maps. Rotation number. Hyperbolic Theory. Stable and Unstable Manifolds. Hartman-Grobman. The Smale Horseshoe. How a horseshoe arises from the transversal intersection of stable and unstable manifolds. Bernoulli shifts buried in the horseshoe. Coin flipping. Logistic map. The pendulum. Geodesic Flows. The constant curvature cases. Integrable systems. Perturbations. Of integrable systems. Of constant curvature geodesic flows. Time permitting. Statement of Anosov theorem. Statement of KAM. Relations between topology and dynamics. Index theorems of Poincare-Hopf. Of Lefshetz. Counting orbits. Lyapanov exponents. Entropy. Ergodic theory. Arnol'd conjecture.

By Week.

1. Examples, starting on the circle: a gradient flow (1st order) and the pendulum (2nd order), going to N-body and manifolds. Initial defintions. Basic ODEs:Well-posedness. Straightening lemma. Flows. Maps. Poincare section, perhaps.

2. What happens with the perturbed pendulum near the homoclinic point? [REF: Guckenheimer-Holmes] Begin hyperbolic theory. Some Linear theory and linearization. Poincare Return Map in detail, and its "inverse" suspension. Conjugation and semi-conjugation.

Perhaps: More examples: rotation on circle. Linear flow on torus. Cat Map, 1. Henon. Cat Map. Fixed points and periodic orbits.

3. Hyperbolic Theory, aiming for Smale's theorem on homoclinic tangles. [REF: Zehnder]. Hartman-Grobman [REF: Hartman]. Stable and Unstable Manifolds [REF: Robinson, or Zehnder] Hartman-Grobman.

The Smale Horseshoe. How it arises as the intersection of stable and unstable manifolds.

Model example: Bernoulli shifts and the Horseshoe. Coin flipping. Logistic map. Smale's theorem, a sketch of proof, [REF: Zehnder] The pendulum, again [REF: Guckenheimer-Holmes]

4 . Geodesic Flows. The (integrable) constant curvature cases, $K < 0, K = 0, K > 0$. Their perturbations. $K < 0$: Anosovness and stability of maps in the singularity theory sense. $K \geq 0$: KAM and islands of chaos.

5. Ergodic theory [REF: Mañe; wiki, EMS II, Dynamical Systems II, Sinai (ed); Halmos; Reed and Simon] A. Basic defintions and examples. Poincare recurrence!. Spectral invariants

6. Entropy, measure theoretic. Ornstein's theorem for the entropy of Bernoulli shifts. Entropy, topological.

7. Topology associated to fixed points or periodic orbits. Lefshetz. Poincare-Hopf. Arnol'd. Orbit counts and entropy.

8. Synopsis. Devaney's definitions of 'chaos'. Lyapanov exponents.

9. Any energy left? - N-body. More on geodesic flows. Contreras paper. Paternain book....