Weeks 1-3. Definition of a Riemann surface. Examples: The Riemann sphere. The Riemann surface of a multi-valued function Eg. $w = \sqrt{P(z)}$.Algebraic curves: $w^2 = P(z)$, both affine and projective.

Uniformization. The three simply connected Riemann surfaces. Riemann surfaces as quotients. The torus. Lattices in $\mathbb{C}$. Poincaré's work: uniformization via lattices in $SU(1,1)$.

Complex projective n-space.

***** I will miss lectures 2, 3 and 4. We will arrange 3 makeup lectures, preferably on Fridays. **********

week 4. The AMAZING SYTHESIS (Mumford's phrase). (1) [Analysis] Compact Riemann surfaces (2) [Algebraic geometry] smooth 1 dimensional projective varieties of $\mathbb{P}^n = \mathbb{C}\mathbb{P}^n$. (3)[Algebra] Field extensions of transcendence degree one. Sketch equivalences.

Genus: Hurwitz formula for. Miranda, p. 52. As a topological *and* birational invariant. $g = \binom{d-1}{2}$ for degree d smooth curves in $\mathbb{P}^2$. Hurwitz formula for isomorphisms of a cpt Riem surf. (Miranda, p. 84).

week 5 -6. Divisors. Linear equivalence of. Ordering of. Equivalence with line bundles and with invertible sheaves. Examples on $\mathbb{P}^1$, on $\mathbb{P}^n$. on torii (elliptic curves). $\mathbb{P}^1$ classification of. *Constructions of* pull back. hyperplane section. *Modular functions* as sections of a line bundle. *The canonical line bundle* K whose sections are holomorphic one-forms.

Using divisors. Ample line bundles yield projective embeddings.

Riemann-Roch, statement 1. Statement 2. Statement 3.

week 7. Riemann-Roch applications, ct'd.

week 8-10. The Jacobian of a Riem surface. The Abel Jacobi map. Periods. Riemann bilinear identities. Families of Riemann surfaces. Moduli space(s) of Riemann surfaces.

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primary texts : (week 1) Milnor: ch. 1 of 'Dynamics in One Complex Variable'

(week 4 and intermittently:) Mumford: 'Curves and their Jacobians'

Miranda: 'Algebraic curves and Riemann surfaces' , throughout.

secondary texts: Brieskorn and Knorrer: Plane Algebraic Curves

Clemens: A Scrapbook of Complex Curve Theory

Kirwan : Complex Algebraic Curves.

Reid: Undergraduate Algebraic Geometry

Hulek: Elementary Algebraic Geometry

tertiary texts : Abhyankar: Algebraic Geometry for Scientists and Engineers.

Ahlfors: Complex Analysis

Chern: Complex Manifolds without Potential Theory.

Hartshorne: Algebraic Geometry

Gerd Fischer: Plane Algebraic Curves

Milnor: Dynamics of One Complex Variable (ch. 1)

C.T.C. Wall: Singular Points of Plane Curves.

Poincaré: Theory of Fuchsian Groups. Acta Math. 1882. Available in Stillwell's 'Sources of Hyperbolic Geometry'.

Gunning: Lectures on Riemann Surfaces.

Weyl: The idea of a Riemann Surface.