Riemann surface HW, WINTER 2022

Homework Assignment I B :

1. If S is a compact RS show that every holomorphic function on S is constant. Hint: Use the maximum modulus principle.

2. Prove that on a Riemannian surface (as opposed to a Riemann surface) the Laplacian Δ transforms conformally: if $\Delta_g f = \psi \Delta_{g'} f$ where $g' = e^u g$ is conformally equivalent to g and $\Delta_g, \Delta_{g'}$ are the Laplacians for these two metrics. Compute the scalar function $\psi = \psi(u)$.

3. Let $X = CP^1 = S^2$, viewed as a Riemann surface.

Show that there is no holomorphic map $X \to X$ which is degree 2 and branched over 3 points.