

This is the second course in the geometry-topology graduate sequence 208-211 here at UCSC. The main subject is the exterior differential calculus and the integration and differentiation of  $k$ -forms. The main theorem is Stokes' theorem. Along the way we will touch on Lie groups, metrics, the tensor calculus, parallelizability, vector bundles, metrics, and the de Rham theorem.

Format. HW will be due Mondays. On Wednesday students will be assigned to present one of the HW solutions for the HW due the following Monday. There will be one midterm. There will be some work done in class.

Lecture-by-lecture schedule, along with HW.

1.  $k$ -forms heuristically. One-forms precisely. Invariantly. In coordinates. Pictures.  $f^*$  [needed for HW1.] Integrating one-forms. coordinate version. Algebra of forms. Calculus on forms. Differentiating, integrating, and pulling back forms. Stokes theorem.

2. One- Forms: invariant version. Pictorial version. Cotangent bundle.  $d$ .  $k$ -forms. Calculus on manifolds.

3. *HW 1 Due.* Begin: Vector bundles. Examples. Tautological bundles. Tangent bundles. Cotangent bundles. Duality. Functorial linear algebra constructions applied to vector bundles.

4. Vector bundles continued. Statement of Poincare-Hopf. Some topology around bundles. (Poincare-Hopf; obstruction theory,...(?))

5. *HW 2 Due.*  $k$ -forms. Exterior differential calculus. Basic operations  $d$ ,  $\wedge$ ,  $i_v$  or  $v$ .  $L_v$ .  $F^*$ .

6. Integrating  $k$ -forms. Focus on 2-forms. Statement of Stokes' theorem.

7. *HW 3 Due.* The general tensor calculus. Splitting of  $T^*M \otimes T^*M$  into '2-forms' and 'metrics'. Poisson tensors.

8. Lie groups.  $SO(3)$ ,  $SU(2)$ .  $O(n)$ .  $Sp(n)$ .  $U(n)$ . Framings. Coframings. Structure constants. Parallelizable manifolds.

9. *Midterm handed out.* Cartan's magic formula. Other identities in the exterior differential calculus.

10. *Discussion of Midterm Metrics.* Examples from surface theory. Structure equations for Riemannian surfaces: getting the Gauss curvature.

11. *Midterm Due.* . Lie derivative, generally. Cartan's magic formula. Moser's method (?)

12. Dual Frobenius. Application to sub-algebras and subgroups. Chow-Rashevskii ( Anti-Frobenius) and its dual.

13. *HW 4 Due.* deRham theorem.

14. Applications of deRham theorem. Ingredients of cohomology.

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